4.4 Increasing and Decreasing Function

4.4.1 Definition

(1) **Strictly increasing function :** A function f(x) is said to be a strictly increasing function on (a, b), if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ for all x_1 , $x_2 \in (a, b)$.

Thus, f(x) is strictly increasing on (*a*, *b*), if the values of f(x) increase with the increase in the values of *x*.

(2) **Strictly decreasing function :** A function f(x) is said to be a strictly decreasing function on (a,b), if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ for all

 $x_1, x_2 \in (a, b)$. Thus, f(x) is strictly decreasing on (a, b), if the values of f(x) decrease with the increase in the values of x.

 $f(x_1 f(x$



(d) x < 2

Solution: (c) $f(x) = (x-1)^2 - 1$

Hence decreasing in *x* < 1

(c) *x* < 1



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[AMU 1999]

nor



Alternative method: f'(x) = 2x - 2 = 2(x - 1)To be decreasing, $2(x-1) < 0 \implies (x-1) < 0 \implies x < 1$. $2x^3 + 18x^2 - 96x + 45 = 0$ is an increasing function when Example: 3 (b) $x < -2, x \ge 8$ (a) $x \le -8, x \ge 2$ (c) $x \le -2, x \ge 8$ (d) $0 < x \le -2$ **Solution:** (a) $f'(x) = 6x^2 + 36x - 96 > 0$, for increasing $\Rightarrow f'(x) = 6(x+8)(x-2) \ge 0 \Rightarrow x \ge 2, x \le -8.$ The function x^x is increasing, when Example: 4 [MP PET 2003] (a) $x > \frac{1}{x}$ (b) $x < \frac{1}{x}$ (c) x < 0(d) For all real x **Solution:** (a) Let $y = x^x \Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$; For $\frac{dy}{dx} > 0$ $x^{x}(1 + \log x) > 0 \implies 1 + \log x > 0 \implies \log_{e} x > \log_{e} \frac{1}{e}$ For this to be positive, x should be greater than $\frac{1}{e}$.

4.4.2 Monotonic Function

A function f(x) is said to be monotonic on an interval (a, b) if it is either increasing or decreasing on (a, b).

(1) **Monotonic increasing function :** A function is said to be a monotonic increasing function in defined interval if, u

 $x_1 > x_2 \implies f(x_1) \ge f(x_2)$ or $x_1 > x_2 \Longrightarrow f(x_1) \le f(x_2)$ or $x_1 < x_2 \Longrightarrow f(x_1) \le f(x_2)$ or $x_1 < x_2 \Longrightarrow f(x_1) \ge f(x_2)$



(2) **Monotonic decreasing function:** A function is said to be a monotonic decreasing function in defined interval, if $x_1 > x_2 \implies f(x_1)$ *y y*





Example: 5 The function $f(x) = \cos x - 2px$ is monotonically decreasing for

[MP PET 2002]

	(a) $p < \frac{1}{2}$	(b) $p > \frac{1}{2}$	(c) <i>p</i> < 2	(d) $p > 2$					
Solution: (b)	f(x) will be monotonical	by decreasing, if $f'(x) < 0$).						
	$\Rightarrow f'(x) = -\sin x - 2p < 0 \Rightarrow$	$\frac{1}{2}\sin x + p > 0 \implies p > \frac{1}{2}$	$[\because -1 \le \sin x \le 1]$						
Example: 6	If $f(x) = x^5 - 20x^3 + 240x$,	then $f(x)$ satisfies which	ch of the following	[Kurukshetra CEE 1996]					
	(a) It is monotonically d	ecreasing everywhere	(b) It is monotonically o	lecreasing only in $(0,\infty)$					
	(c) It is monotonically in	ncreasing every where	(d) It is monotonically i	ncreasing only in $(-\infty, 0)$					
Solution: (c)	$f'(x) = 5x^4 - 60x^2 + 240 =$	$5(x^4 - 12x^2 + 48) = 5[(x^2 - 12x^2 + 48)] = 5[(x^2 - 12x^2 + 12x^2$	$(6)^2 + 12]$						
	$\implies f'(x) > 0 \forall x \in R$								
	<i>i.e.</i> , $f(x)$ is monotonically	y increasing everywher	e.						
Example: 7 real <i>x</i> , are	The value of a for which	the function $(a+2)x^3 - 3$	$Bax^2 + 9ax - 1$ decrease more	notonically throughout for all					
				[Kurukshetra CEE 2002]					
	(a) <i>a</i> < -2	(b) $a > -2$	(c) $-3 < a < 0$	(d) $-\infty < a \le -3$					
Solution: (d)	If $f(x) = (a+2)x^3 - 3ax^2 + 9ax^3 - 3ax^2 + 9ax^3 - 3ax^2 + 9ax^3 - 3ax^2 + 9ax^3 - 3ax^3 -$	ax-1 decreases monoto	onically for all $x \in R$, then	$f'(x) \le 0$ for all $x \in R$					
	$\Rightarrow 3(a+2)x^2 - 6ax + 9a \le 0$	for all $x \in R \implies (a+2)x^2$	$a^2 - 2ax + 3a \le 0$ for all $x \in R$	for all $x \in R$					
	$\Rightarrow a+2<0$ and discrimin	ant ≤ 0	$\Rightarrow a < -2$ and $-8a^2 - 24a$	≤ 0					
	$\Rightarrow a < -2 \text{ and } a(a+3) \ge 0 =$	$a < -2$ and $a \le -3$ or $a \le -3$	$a \ge 0 \Rightarrow a \le -3 \Rightarrow -\infty < a \le -3$	3					
Example: 8	Function $f(x) = \frac{\lambda \sin x + 6 \operatorname{c}}{2 \sin x + 3 \operatorname{c}}$	$\frac{\cos x}{\cos x}$ is monotonic incre	easing if						
	(a) $\lambda > 1$	(b) $\lambda < 1$	(c) $\lambda < 4$	(d) $\lambda > 4$					
Solution: (d)	The function is monotoni	ic increasing if, $f'(x) > 0$							
	$\Rightarrow \frac{(2\sin x + 3\cos x)(\lambda\cos x - 6\sin x)}{(2\sin x + 3\cos x)^2} - \frac{(\lambda\sin x + 6\cos x)(2\cos x - 3\sin x)}{(2\sin x + 3\cos x)^2} > 0$								
	$(2\sin x + 3\cos x)^2$	$(2\sin x +$	$\frac{1}{3\cos x^2} > 0$						

4.4.3 Necessary and Sufficient Condition for Monotonic Function

In this section we intend to see how we can use derivative of a function to determine where it is increasing and where it is decreasing

all

(1) **Necessary condition :** From figure we observe that if f(x) is an increasing function on (*a*,

b), then tangent at every point on the curve y = f(x) makes an acute angle θ with the positive direction of *x*-axis.

$$\therefore \quad \tan \theta > 0 \Rightarrow \frac{dy}{dx} > 0 \text{ or } f'(x) > 0 \quad \text{for}$$

 $x \in (a, b)$

It is evident from figure that if f(x) is a



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decreasing function on (*a*, *b*), then tangent at every point on the curve y = f(x) makes an obtuse angle θ with the positive direction of *x*-axis.

:
$$\tan \theta < 0 \Rightarrow \frac{dy}{dx} < 0$$
 or $f'(x) < 0$ for all $x \in (a, b)$.

Thus, f'(x) > 0(< 0) for all $x \in (a, b)$ is the necessary condition for a function f(x) to be increasing (decreasing) on a given interval (a, b). In other words, if it is given that f(x) is increasing (decreasing) on (a, b), then we can say that f'(x) > 0 (< 0).

(2) **Sufficient condition : Theorem :** Let *f* be a differentiable real function defined on an open interval (*a*, *b*).

(a) If f'(x) > 0 for all $x \in (a, b)$, then f(x) is increasing on (a, b).

(b) If f'(x) < 0 for all $x \in (a, b)$, then f(x) is decreasing on (a, b).

Corollary : Let f(x) be a function defined on (a, b).

(a) If f'(x) > 0 for all $x \in (a, b)$, except for a finite number of points, where f'(x) = 0, then f(x) is increasing on (a, b).

(b) If f'(x) < 0 for all $x \in (a, b)$, except for a finite number of points, where f'(x) = 0, then f(x) is decreasing on (a, b).

Example: 9 The function
$$f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$$
 is
(a) Increasing on $[0, \infty)$ (b) Decreasing on $[0, \infty)$
(c) Decreasing on $\left[0, \frac{\pi}{e}\right)$ and increasing on $\left[\frac{\pi}{e}, \infty\right)$ (d) Increasing on $\left[0, \frac{\pi}{e}\right)$ and
decreasing on $\left[\frac{\pi}{e}, \infty\right)$
Solution: (b) Let $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$
 $\therefore f'(x) = \frac{\ln(e + x) \times \frac{1}{\pi + x} - \ln(\pi + x) \frac{1}{e + x}}{(\ln(e + x))^2} = \frac{(e + x)\ln(e + x) - (\pi + x)\ln(\pi + x)}{(\ln(e + x))^2 \times (e + x)(\pi + x)}$
 $\Rightarrow f'(x) < 0$ for all $x \ge 0$ { $\because x > e$ }. Hence, $f(x)$ is decreasing in $[0, \infty)$.
Example: 10 Which of the following is not a decreasing function on the interval $\left(0, \frac{\pi}{2}\right)$
(a) $\cos x$ (b) $\cos 2x$ (c) $\cos 3x$ (d) $\cot x$
Solution: (c) Obviously, here $\cos 3x$ in not decreasing in $\left(0, \frac{\pi}{2}\right)$ because $\frac{d}{dx} \cos 3x = -3 \sin 3x$.
But at $x = 75^{\circ}$, $-3 \sin 3x > 0$. Hence the result.
Example: 11 The interval of increase of the function $f(x) = x - e^{x} + \tan\left(\frac{2\pi}{7}\right)$ is
(a) $(0, \infty)$ (b) $(-\alpha, 0)$ (c) $(1, \infty)$ (d) $(-\alpha, -1)$

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[IIT Screening 2001]

Solution: (b, d) We have $f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right) \Rightarrow f'(x) = 1 - e^x$ For f(x) to be increasing, we must have $f'(x) > 0 \Rightarrow 1 - e^x > 0 \Rightarrow e^x < 1 \Rightarrow x < 0 \Rightarrow x \in (-\infty, 0) \Rightarrow (-\infty, -1) \subseteq (-\infty, 0)$

4.4.4 Test for Monotonicity

(1) At a point : (i) Function f(x) will be monotonic increasing in domain at a point if and only if, f'(a) > 0

(ii) Function f(x) will be monotonic decreasing in domain at a point if and only if, f'(a) < 0.

(2) In an interval : Function f(x), defined in [a, b] is

(i) Monotonic increasing in (a, b) if, $f'(x) \ge 0$, a < x < b

(ii) Monotonic increasing in [a, b] if, $f'(x) \ge 0$, $a \le x \le b$

(iii) Strictly increasing in [a, b], if, f'(x) > 0, $a \le x \le b$

(iv) Monotonic decreasing in (a, b), if, $f'(x) \le 0$, a < x < b

(v) Monotonic decreasing in [a, b], if, $f'(x) \le 0$, $a \le x \le b$

(vi) Strictly decreasing in [a, b], if, f'(x) < 0, $a \le x \le b$

Example: 12 $f(x) = xe^{x(1-x)}$ then f(x) is

(a) Increasing on $\left[\frac{-1}{2}, 1\right]$ (b) Decreasing on R (c) Increasing on R (d) Decreasing on $\left[\frac{-1}{2}, 1\right]$

Solution: (a) $f'(x) = e^{x(1-x)} + x \cdot e^{x(1-x)} \cdot (1-2x) = e^{x(1-x)} \{1 + x(1-2x)\} = e^{x(1-x)} \cdot (-2x^2 + x + 1)$

Now by the sign-scheme for $-2x^2 + x + 1$



 $f'(x) \ge 0$, if $x \in \left[-\frac{1}{2}, 1\right]$, because $e^{x(1-x)}$ is always positive. So, f(x) is increasing on $\left|-\frac{1}{2}, 1\right|$.

Example: 13x tends 0 to π then the given function $f(x) = x \sin x + \cos^2 x$ is(a) Increasing(b) Decreasing(c) Neither increasing nor decreasing(d) None of these

Solution: (b) $f(x) = x \sin x + \cos x + \cos^2 x$

 $\therefore f'(x) = \sin x + x \cos x - \sin x - 2 \cos x \sin x = \cos x (x - 2 \sin x)$

Hence $x \to 0$ to π , then $f'(x) \le 0$, *i.e.*, f(x) is decreasing function.

4.4.5 Properties of Monotonic Function

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(1) If f(x) is strictly increasing function on an interval [*a*, *b*], then f^{-1} exists and it is also a strictly increasing function.

(2) If f(x) is strictly increasing function on an interval [*a*, *b*] such that it is continuous, then f^{-1} is continuous on [f(a), f(b)]

(3) If f(x) is continuous on [a, b] such that $f'(c) \ge 0(f'(c) > 0)$ for each $c \in (a,b)$, then f(x) is monotonically (strictly) increasing function on [a, b].

(4) If f(x) is continuous on [a, b] such that $f'(c) \le 0(f'(c) < 0)$ for each $c \in (a,b)$, then f(x) is monotonically (strictly) decreasing function on [a, b]

(5) If f(x) and g(x) are monotonically (or strictly) increasing (or decreasing) functions on [*a*, *b*], then gof(x) is a monotonically (or strictly) increasing function on [*a*, *b*]

(6) If one of the two functions f(x) and g(x) is strictly (or monotonically) increasing and other a strictly (monotonically) decreasing, then gof(x) is strictly (monotonically) decreasing on [a, b].

Example: 14 The interval in which the function $x^2 e^{-x}$ is non decreasing, is

(a) $(-\infty, 2]$ (b) [0, 2] (c) $[2, \infty)$ (d) None of these

Solution: (b) Let $f(x) = x^2 e^{-x}$

$$\Rightarrow \frac{dy}{dx} = 2xe^{-x} - x^2e^{-x} = e^{-x}(2x - x^2)$$

Hence $f'(x) \ge 0$ for every $x \in [0, 2]$, therefore it is non-decreasing in [0, 2].

Example: 15 The function $\sin^4 x + \cos^4 x$ increase if

(a)
$$0 < x < \frac{\pi}{8}$$
 (b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$ (c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

Solution: (b) $f(x) = \sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$$= 1 - \frac{4\sin^2 x \cos^2 x}{2} = 1 - \frac{\sin^2 2x}{2} = 1 - \frac{1}{4}(2\sin^2 2x)$$
$$= 1 - \left(\frac{1 - \cos 4x}{4}\right) = \frac{3}{4} + \frac{1}{4}\cos 4x$$

Hence function f(x) is increasing when f'(x) > 0

$$f'(x) = -\sin 4x > 0 \implies \sin 4x < 0$$

Hence $\pi < 4x < \frac{3\pi}{2}$ or $\frac{\pi}{4} < x < \frac{3\pi}{8}$.

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[IIT 1999]



Increasing	and	Decreasing	Function

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Basic Level

The function $x + \frac{1}{x}$ ($x \neq 0$) is a non-increasing function in the interval 1. (a) [-1,1] (b) [0, 1] (c) [-1,0] (d) [-1, 2] The interval for which the given function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is decreasing, is 2. (a) (- 2, 3) (b) (2, 3) (c) (2, -3) (d) None of these If $f(x) = \sin x - \frac{x}{2}$ is increasing function, then [MP PET 1987] 3. (b) $-\frac{\pi}{3} < x < 0$ (c) $-\frac{\pi}{3} < x < \frac{\pi}{3}$ (d) $x = \frac{\pi}{2}$ (a) $0 < x < \frac{\pi}{3}$ If the function $f: R \to R$ be defined by $f(x) = \tan x - x$, then f(x)4. (a) Increases (b) Decreases (c) Remains constant (d) Becomes zero

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5۰	$2x^3 - 6x + 5$ is an increase	easing function if		[UPSEAT 2003]		
	(a) $0 < x < 1$	(b) $-1 < x < 1$	(c) $x < -1$ or $x > 1$	(d) $-1 < x < -1/2$		
6.	The function $f(x) = 1 - 1$	$x^3 - x^5$ is decreasing for		[Kerala (Engg.) 2002]		
	(a) $1 \le x \le 5$	(b) $x \le 1$	(c) $x \ge 1$	(d) All values of <i>x</i>		
7.	For which interval, th	e given function $f(x) = -2x^3 - 9x^3$	$x^2 - 12x + 1$ is decreasing	[MP PET 1993]		
	(a) (−2,∞)	(b) (-2,-1)	(c) $(-\infty, -1)$	(d) $(-\infty, -2)$ and $(-1, \infty)$		
8.	The function $f(x) = \tan x$	x - x		[MNR 1995]		
	(a) Always increases		(b) Always decreases			
	(c) Never decreases		(d) Sometimes increase	es and sometimes decreases		
9.	If $f(x) = kx^3 - 9x^2 + 9x +$ Kurukshetra CEE 2002]	3 is monotonically increasing	g in each interval, then	[Rajasthan PET 1992;		
	(a) <i>k</i> < 3	(b) $k \le 3$	(c) $k > 3$	(d) None of these		
10.	The least value of <i>k</i> fo	or which the function $x^2 + kx + 1$	is an increasing function in	the interval $1 < x < 2$ is		
	(a) - 4	(b) - 3	(c) - 1	(d) - 2		
11.	The function $f(x) = x + $	$\cos x$ is				
	(a) Always increasing	Ş	(b) Always decreasing			
	(c) Increasing for cer	tain range of <i>x</i>	(d)	None of these		
12.	The function $f(x) = x^2$	is increasing in the interval				
	(a) (-1,1)	(b) (−∞,∞)	(C) (0,∞)	(d) (-∞,0)		
13.	Function $f(x) = x^4 - \frac{x^3}{3}$	is				
	(a) Increasing for $x >$	$\frac{1}{4}$ and decreasing for $x < \frac{1}{4}$	(b) Increasing for ever	y value of x		
	(c) Decreasing for eve	ery value of <i>x</i>	(d)	None of these		
14.	The function $y = 2x^3 -$ PET 1994; Rajasthan PE	$9x^2 + 12x - 6$ is monotonic decree ET 1996]	asing when	[MP		
	(a) $1 < x < 2$	(b) $x > 2$	(c) <i>x</i> < 1	(d) None of these		
15.	The interval in which	the $x^2 e^{-x}$ is non-decreasing, is				
	(a) (-∞,2]	(b) [0, 2]	(c) [2,∞)	(d) None of these		
16.	The function $\frac{1}{1+x^2}$ is	decreasing in the interval				
	(a) (-∞,-1]	(b) (-∞,0]	(C) [1,∞)	(d) (0,∞)		
17.	The function $\sin x - bx$	+c will be increasing in the int	erval $(-\infty,\infty)$ if			
	(a) <i>b</i> ≤1	(b) $b \le 0$	(c) $b < -1$	(d) $b \ge 0$		
18.	In the interval [0, 1],	the function $x^2 - x + 1$ is				
	(a) Increasing		(b) Decreasing			

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			Appl	ication of Derivatives 227						
	(c) Neither increasing no	r decreasing	(d) None of these							
19.	$f(x) = x^3 - 27x + 5$ is an inc.	reasing function, when		[MP PET 1995]						
	(a) $x < -3$	(b) $ x > 3$	(c) $x \le -3$	(d) x <3						
20.	For the every value of <i>x</i> the transformed term of <i>x</i> the transformed term of <i>x</i> the transformed term of the term of	the function $f(x) = \frac{1}{5^x}$ is								
	(a) Decreasing		(b) Increasing							
	(c) Neither increasing no	r decreasing	(d) Increasing for $x > 0$ a	and decreasing for $x < 0$						
21.	In which interval is the gi	ven function $f(x) = 2x^3 - 15x^2 + $	-36x+1 is monotonically dec	creasing						
	(a) [2, 3]	(b) (2, 3)	(c) (-∞,2)	(d) (3,∞)						
22.	The interval of the decrea	sing function $f(x) = x^3 - x^2 - x^2$	-4 is							
	(a) $\left(\frac{1}{3}, 1\right)$	(b) $\left(-\frac{1}{3},1\right)$	(c) $\left(-\frac{1}{3},\frac{1}{3}\right)$	$(\mathbf{d}) \left(-1,-\frac{1}{3}\right)$						
23.	Let $f(x) = x^3 + bx^2 + cx + d, 0$	$< b^2 < c$. Then f		[IIT JEE Screening 2004]						
	(a) Is bounded	(b) Has a local maxima	(c) Has a local minima	(d) Is strictly increasing						
24.	The function $f(x) = x^3 - 3x^3$	$x^2 - 24x + 5$ is an increasing fun	nction in the interval given below							
	(a) $(-\infty, -2) \cup (4,\infty)$	(b) (−2,∞)	(c) (-2,4) (d) (-∞,4)							
25.	Which one is the correct statement about the function $f(x) = \sin 2x$									
	• Which one is the correct statement about the function $f(x) = \sin 2x$ (a) $f(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$ and decreasing in $\left(\frac{\pi}{2}, \pi\right)$									
	(b) $f(x)$ is decreasing in ($\left(0,\frac{\pi}{2}\right)$ and increasing in $\left(\frac{\pi}{2},\pi\right)$								
	(c) $f(x)$ is increasing in $\left(\int_{-\infty}^{\infty} f(x) dx \right) = \int_{-\infty}^{\infty} f(x) dx$	$(0,\frac{\pi}{4})$ and decreasing in $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$								
	(d) The statement (a), (b) and (c) are all correct								
26.	If $f(x) = x^3 - 10x^2 + 200x - 10x^2 + 200x^2 + 10x^2 + 10x^2 + 10x^2 + 10x^2 + 10x^2 + 10x^2 $	0, then		[Kurukshetra CEE 1998]						
	(a) $f(x)$ is decreasing in]	$-\infty,10$] and increasing in [10, \circ	p(b) f(x) is increasing in	n] $-\infty$,10]and decreasing in						
	$[10,\infty]$	hughout roal line	(d) $f(x)$ is decreasing three	hughout roal line						
	(c) $f(x)$ is increasing the	(u^2) ((u)	(d) $f(x)$ is decreasing throughout real line							
27.	If <i>f</i> is a strictly increasing	function, then $\lim_{x \to 0} \frac{f(x) - f(x)}{f(x) - f(0)}$	is equal to							
	(a) 0	(b) 1	(c) - 1	(d) 2						
28.	Function $x^3 - 6x^2 + 9x + 1$ is	s monotonic decreasing when		[Rajasthan PET 1991]						
	(a) $1 < x < 3$	(b) $x < 3$	(c) $x > 1$ (d) $x > 3$ or $x < 1$							
29.	The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^3$	$\frac{1}{2}x^2 - 6x + 8$ is decreasing in the	ne interval							
	(a) <i>x</i> < -3	(b) $x > 2$	(c) $-3 < x < 2$	(d) None of these						

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30.	The function $f(x) = 2\log(x - x)$	2) – x^2 + 4 x + 1 increases in the	interval							
	(a) (1, 2)	(b) (2, 3)	(c) $(-\infty, -1)$	(d) (2, 4)						
31.	The function $f(x) = \frac{ x }{x} (x \neq x)$	0), $x > 0$ is								
	(a) Monotonically decreas	ing (b)	Monotonically increasing	(c) Constant function (d)						
32.	In the following decreasin	g function is								
	(a) ln <i>x</i>	(b) $\frac{1}{ x }$	(c) $e^{1/x}$	(d) None of these						
33.	If $f(x) = kx - \sin x$ is monoto	nically increasing, then								
	(a) <i>k</i> > 1	(b) $k > -1$	(c) <i>k</i> < 1	(d) <i>k</i> < -1						
		Advance L	evel							
34.	The function <i>f</i> defined by	$f(x) = (x+2)e^{-x}$ is		[IIT Screening 1994]						
	(a) Decreasing for all <i>x</i>		(b) Decreasing in $(-\infty, -1)$ and increasing in $(-1, \infty)$							
	(c) Increasing for all <i>x</i>		(d) Decreasing in $(-1,\infty)$ and increasing in $(-\infty, -1)$							
35.	The value of <i>a</i> in order th	at $f(x) = \sqrt{3} \sin x - \cos x - 2ax + \frac{1}{2}$	x+b decreases for all real values of x, is given by							
	(a) <i>a</i> < 1	(b) $a \ge 1$	(c) $a \ge \sqrt{2}$	(d) $a < \sqrt{2}$						
36.	The interval in which the	function x^3 increases less rap	bidly then $6x^2 + 15x + 5$, is							
	(a) $(-\infty, -1)$	(b) (-5,1)	(c) (-1,5)	(d) $(5,\infty)$						
37.	Let $f(x) = \int e^{x} (x-1)(x-2) dx$. Then <i>f</i> decreases in the inter	rval							
	(a) (−∞,−2)	(b) (-2,-1)	(c) (1, 2)	(d) $(2, +\infty)$						
38.	If $f(x) = 2x + \cot^{-1} x + \log(\sqrt{1 + \cos^{-1} x})$	$+x^2 - x$), then $f(x)$								
	(a) Increases in $[0,\infty)$		(b) Decreases in $[0,\infty)$							
	(c) Neither increases nor	decreases in $(0,\infty)$	(d) Increases in $(-\infty,\infty)$							
39.	The function $\frac{(e^{2x}-1)}{(e^{2x}+1)}$ is			[Roorkee 1998]						
	(a) Increasing	(b) Decreasing	(c) Even	(d) Odd						
40.	The function $\frac{a \sin x + b \cos x}{c \sin x + d \cos x}$	is decreasing if		[Rajasthan PET 1999]						
	(a) $ad - bc > 0$	(b) $ad - bc < 0$	(c) $ab - cd > 0$	(d) $ab - cd < 0$						
41.	If $f(x) = \sin x - \cos x$, $0 \le x \le$	2π the function decreasing in		[UPSEAT 2001]						
	(a) $\left[\frac{5\pi}{6},\frac{3\pi}{4}\right]$	(b) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$	(c) $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$	(d) None of these						

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Application of Derivatives 229 **42.** If $f(x) = \frac{1}{x+1} - \log(1+x), x > 0$ then f is [Rajasthan PET 2002] (a) An increasing function (b) A decreasing function (c) Both increasing and decreasing function (d) None of these The function $f(x) = x^{1/x}$ is 43. [AMU 2002] (a) Increasing in $(1,\infty)$ (b) Decreasing in $(1,\infty)$ (d) Decreasing in (1,e) increasing in (e,∞) (c) Increasing in (1,e), decreasing in (e,∞) **44.** The length of the longest interval, in which the function $3 \sin x - 4 \sin^3 x$ is increasing, is (c) $\frac{3\pi}{2}$ (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (d) π The function $f(x) = 1 - e^{-x^2/2}$ is 45. (a) Decreasing for all x (b) Increasing for all x (c) Decreasing for x < 0 and increasing for x > 0(d) Increasing for x < 0 and decreasing for x > 0**46.** The function $\sin x - \cos x$ is increasing in the interval (c) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (a) $\left| \frac{3\pi}{4}, \frac{7\pi}{4} \right|$ (b) $\left[0, \frac{3\pi}{4}\right]$ (d) None of these **47.** On the interval $\left(0, \frac{\pi}{2}\right)$, the function log sin *x* is (a) Increasing (b) Decreasing (d) None of these (c) Neither increasing nor decreasing **48.** For all real values of x, increasing function f(x) is [MP PET 1996] (c) x^{3} (a) x^{-1} (b) x^2 (d) x^4 The function which is neither decreasing nor increasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is 49. (b) tan x (c) x^2 (d) |x-1|(a) cosec x For every value of *x*, function $f(x) = e^x$ is 50. (a) Decreasing (b) Increasing (c) Neither increasing nor decreasing (d) None of these Consider the following statements S and R51. S: Both sin x and cos x are decreasing functions in $\left(\frac{\pi}{2},\pi\right)$ R: If a differentiable function decreases in (a, b) then its derivative also decrease in (a, b) Which of the following is true (a) Both S and R are wrong (b) Both S and R are correct but R is not the correct explanation for S

- (c) *S* is correct and *R* is the correct explanation for *S*
- (d) *S* is correct and *R* is wrong

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52. If f'(x) is zero in the interval (a,b) then in this interval it is (a) Increasing function (b) Decreasing function (c) Only for a > 0 and b > 0 is increasing function (d) None of these The function $\frac{x-2}{x+1}$, $(x \neq -1)$ is increasing on the interval 53. (a) (-∞,0] **(b)** [0,∞) (c) R (d) None of these 54. If *f* and *g* are two decreasing functions such that *fog* exists, then *fog* Is a decreasing function (a) Is an increasing function (b) (c) Is neither increasing nor decreasing (d) None of these The function $f(x) = \cos(\pi / x)$ is increasing in the interval 55. (b) $\left(\frac{1}{2n+1}, 2n\right)$, $n \in N$ (c) $\left(\frac{1}{2n+2}, \frac{1}{2n+1}\right)$, $n \in N$ (d) None of these (a) $(2n+1, 2n), n \in N$ The set of all values of *a* for which the function $f(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right) x^5 - 3x + \log 5$ decreases for all real *x* is 56. (b) $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup (1,\infty)$ (c) $\left(-3, 5-\frac{\sqrt{27}}{2}\right) \cup (2,\infty)$ (d) $[1,\infty)$ (a) (−∞,∞) The function $f(x) = x\sqrt{ax - x^2}, a > 0$ 57. (a) Increases on the interval $\left(0, \frac{3a}{4}\right)$ (b) Decreases on the interval $\left(\frac{3a}{4}, a\right)$ (c) Decreases on the interval $\left(0, \frac{3a}{4}\right)$ (d) Increases on the interval $\left(\frac{3a}{4}, a\right)$ The function $f(x) = \frac{|x-1|}{x^2}$ is monotonically decreasing on 58. (b) (0, 1) (a) (−2,∞) (c) (0, 1) $\cup (2,\infty)$ (d) $(-\infty,\infty)$ The set of values of a for which the function $f(x) = x^2 + ax + 1$ is an increasing function on [1, 2] is 59. (b) [-4,∞] (c) $[-\infty, -2)$ (d) (-∞,2] (a) (−2,∞) **60.** On which of the following intervals is the function $x^{100} + \sin x - 1$ decreasing (c) $\left(\frac{\pi}{2},\pi\right)$ (a) $\left(0,\frac{\pi}{2}\right)$ (b) (0, 1) (d) None of these If a < 0 the function $f(x) = e^{ax} + e^{-ax}$ is a monotonically decreasing function for values of x given by 61. (a) x > 0(b) x < 0(c) x > 1(d) *x* < 1 **62.** $y = [x(x-3)]^2$ increases for all values of x lying in the interval (a) $0 < x < \frac{3}{2}$ (c) $-\infty < x < 0$ (b) $0 < x < \infty$ (d) 1 < x < 3**63.** The function $f(x) = \frac{\log x}{x}$ is increasing in the interval [EAMCET 1994]

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				Application of Derivatives 231							
	(a) (1,2 <i>e</i>)	(b) (0, <i>e</i>)	(c) (2,2 <i>e</i>)	(d) $\left(\frac{1}{e}, 2e\right)$							
64.	The value of <i>a</i> for which	the function $f(x) = \sin x - \cos x$	s x - ax + b decreases for a	ll real values of x, is given by							
	(a) $a \ge \sqrt{2}$	(b) $a \ge 1$	(c) $a < \sqrt{2}$ (d) $a < 1$								
65.	If the function $f(x) = \cos $	$x \mid -2ax + b$ increases along	the entire number scale,	the range of values of <i>a</i> is given by							
	(a) $a \leq b$	(b) $a = \frac{b}{2}$	(c) $a \leq -\frac{1}{2}$	(d) $a \ge -\frac{3}{2}$							
66.	If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{1}{t}$	$\frac{x}{\operatorname{an} x}$, where $0 < x \le 1$, then	in this interval								
	(a) Both $f(x)$ and $g(x)$ and	re increasing functions	(b) Both $f(x)$ and g	(x) are decreasing function							
	(c) $f(x)$ is an increasing function	function	(d)	g(x) is an increasing							
67.	Let $h(x) = f(x) - (f(x))^2 + (f(x))^2$	(x)) ³ for every real number .	x, then								
	(a) <i>h</i> is increasing when	ever f is increasing and de	creasing whenever <i>f</i> is de	creasing							
	(b) <i>h</i> is increasing when	ever <i>f</i> is decreasing									
	(c) <i>h</i> is decreasing whenever <i>f</i> is increasing										
	(d) Nothing can be said	in general									
68.	If $f(x) = \begin{cases} 3x^2 + 12x - 1 & , \\ 37 - x & , \end{cases}$	$-1 \le x \le 2$ $2 < x \le 3$ then $f(x)$ is		[IIT 1993]							
	(a) Increasing in [-1, 2]	(b) Continuous in [-1, 3	(c) Greatest at $x =$	2 (d) All of these							
69.	If $f'(x) = g(x)(x - \lambda)^2$ wher	e $g(\lambda) \neq 0$ and $g(x)$ is conti	nuous at $x = \lambda$ then funct	tion $f(x)$							
	(a) Increasing near to λ $g(\lambda) > 0$	$ if g(\lambda) > 0 $	(b)	Decreasing near to λ if							
	(c) Increasing near to λ every value of $g(\lambda)$	if $g(\lambda) < 0$	(d)	Increasing near to λ for							
7 0 .	Function $\cos^2 x + \cos^2 \left(\frac{\pi}{3} - \frac{\pi}{3}\right)$	$+x$) - cos $x cos\left(\frac{\pi}{3} + x\right)$ for all	real values of <i>x</i> will be								
	(a) Increasing	(b) Constant	(c) Decreasing	(d) None of these							
71.	Let $Q(x) = f(x) + f(1 - x)$ and	d $f''(x) < 0$ whereas $0 \le x \le$	1 then function $Q(x)$ is decreasing in								
	(a) $\left[\frac{1}{2},1\right]$	(b) $\left[0,\frac{1}{2}\right]$	(c) $\left(\frac{1}{2},1\right)$	(d) (0, 1)							
72.	If $f(x) = \frac{x}{c} + \frac{c}{x}$ for $-5 \le x$	\leq 5, then $f(x)$ is increasing	function in the interval								
	(a) [c, 5]	(b) [0, c]	(c) [c, 0]	(d) [c, c]							
73.	If the domain of $f(x) = \sin x$	$f(x) \text{ is } D = \{x : 0 \le x \le \pi\}$, then	f(x) is								
	(a) Increasing in D		(b) Decreasing in D								

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232	Application of Derivativ	es								
	(c) Decreasing in $\left[0, \frac{\pi}{2}\right]$ are	nd increasing in $\left[\frac{\pi}{2},\pi\right]$	(d) None of these							
74.	If $f(x) = (ab - b^2 - 1)x - \int_0^x (\cos^2 b) dx$	$(\theta + \sin^4 \theta)d\theta$ is a decreasing f	g function of x for all $x \in R$ and $b \in R$, b being independent							
	of <i>x</i> , then									
	(a) $a \in (0, \sqrt{6})$	(b) $a \in (-\sqrt{6}, \sqrt{6})$	(c) $a \in (-\sqrt{6}, 0)$ (d) None of these							
75.	If $f(x) = \frac{p^2 - 1}{p^2 + 1}x^3 - 3x + \log 2$	is a decreasing function of x	in R then the set of possible	e values of p (independent of						
	<i>x</i>) is									
	(a) [-1, 1]	(b) [1,∞)	(c) (-∞,-1]	(d) None of these						
76.	Let $f f(x) = a_5 x^5 + a_4 x^4 + a_3 x^3$	$a_i^3 + a_2 x^2 + a_1 x$, where a_i 's are re-	eal and $f(x) = 0$ has a positive	re root α_0 . Then						
	(a) $f'(x) = 0$ has a root α_1	such that $0 < \alpha_1 < \alpha_0$	(b) $f'(x) = 0$ has at least two real root							
	(c) $f''(x) = 0$ has at least on	ne real roots	(d) None of these							
77.	If <i>a</i> , <i>b</i> , <i>c</i> are real, then $f(x)$	$= \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$ is dec	creasing in							
	(a) $\left(-\frac{2}{3}(a^2+b^2+c^2),0\right)$	(b) $\left(0, \frac{2}{3}(a^2 + b^2 + c^2)\right)$	(c) $\left(\frac{a^2+b^2+c^2}{3},0\right)$	(d) None of these						

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\mathcal{A} nswer Sheet

	Assignment (Basic and Advance Level)																		
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	a	С	a	С	d	d	a	С	d	a	с	a	a	b	d	с	d	b	a
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	b	d	a	С	с	С	a	С	b	С	с	a	d	b	с	с	a,d	a,d	b
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
d	b	С	a	С	b	a	с	a	b	d	d	b	a	d	b	a,b	С	a	d
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77			
b	a	b	a	С	с	a	d	a	b	a	a	d	b	a	a,b,	a			
															С				

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